

THE CESIUM THERMIONIC CONVERTER  
INTERELECTRODE ELECTRON DENSITY DISTRIBUTION  
AS DETERMINED BY RADIATION TRANSPORT

Floyd E. Gelhaus\*

Radio Corporation of America  
Lancaster, Pennsylvania

John P. Howe

Department of Engineering Physics  
Cornell University  
Ithaca, New York

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Abstract

The transport of radiation is shown to be important in determining the density and spatial distribution of electrons and ions in the cesium plasma of a thermionic converter in the arc mode. Reactions used, in addition to absorption and emission of radiation by the  $\text{Cs } ^2\text{P}_{3/2}$  state, are:  $2\text{Cs}(^2\text{P}_{3/2}) \rightarrow \text{Cs}_2^+ + e$ ,  $\text{Cs}_2^+ + e \rightarrow \text{Cs}(^2\text{P}_{3/2}) + \text{Cs}(^2\text{S}_{1/2})$  and electron quenching and excitation of  $\text{Cs } ^2\text{P}_{3/2}$ . These last reactions require consideration of a group of electrons with energy above the  $^2\text{P}_{3/2}$  state, 1.45 eV, as well as larger group with energy below this value. Electron and ion densities are given as a function of position, rate of excitation, and selected values of rate constants.

Introduction

The supposition that the cesium molecule ion may be produced in the interelectrode plasma of an arc-mode thermionic converter results from the 1930 experimental work by Freudenberg<sup>1</sup>. He determined that absorption of the 8521 Å resonance line (1.45 eV) by cesium vapor produced positive ions and further, that charge production by this photoionization process was dependent upon the square of the incident radiation intensity. Freudenberg subsequently proposed that a collision between two atoms in the first excited state ( $6^2\text{P}_{3/2}$ ) results in molecule ion production, according to



The formation of a bound, doubly-excited molecule as a necessary step in the photoionization process was not proven by Freudenberg, although the use of this premise by DeBoer<sup>2</sup>, in offering a set of potential curves for the cesium system, and the present apparent acceptance of it as fact, has led experimenters<sup>3</sup> to the conclusion that the molecule ion is not the dominant converter-plasma ionic specie.

Conversely, others<sup>4, 5, 6</sup> have experimentally demonstrated the important role that radiation plays in determining the positive ion concentration in a cesium plasma and

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have concluded in favor of the molecule ion. In particular, Pollock, et. al.<sup>5</sup>, showed that irradiation of a cesium plasma by a wavelength band which encompassed the first-excited-state resonance lines (8521 Å and 8943 Å), produces an enhancement of the space charge current. In discussing the work of Bensimon<sup>7</sup>, who has shown that photoionization can be important in reducing interelectrode space charge, Hatsopoulos states that<sup>8</sup> "...it should be important to include photon transport phenomena in analyzing a cesium vapor diode operating in the ignited mode."

The acceptance of radiation transport as a prevalent plasma phenomenon necessitates the investigation of its dominance, relative to that of ambipolar diffusion, in determining the spatial distribution of charge in the interelectrode plasma. In general, an equation of the form

$$D_a \frac{d^2 n}{dx^2} + P(n, x) - R(n, x) = 0 \quad (2)$$

must describe the spatial distribution of charge. Here  $D_a$  is the ambipolar diffusion coefficient,  $n$  is the charged particle density,  $P(n, x)$  is the rate of production of charge at  $x$  and  $R(n, x)$  is the corresponding rate of charge removal at that point. Using reasonable values for the rate constants (see Table 1), the model described here predicts, for a Maxwellian plasma with  $kT_e = 0.25$  eV, that

$$P(Cs_2^+, 0.127 \text{ cm}) = R(Cs_2^+, 0.127 \text{ cm}) = 1.2 \times 10^{18} (\text{cm}^3\text{-sec})^{-1}.$$

When the charged particle concentrations are large, any departure from electrical neutrality produces strong restoring forces. Under these conditions,

$D_a \approx 2D(Cs_2^+)$ ,  $D(Cs_2^+) = (kT_g/e) \mu_o (Cs_2^+)$  and the diffusion term can be approximated by

$$D_a \frac{d^2 n}{dx^2} = (2kT_g/e) \mu_o (Cs_2^+) \frac{[Cs] [Cs_2^+]}{\Lambda^2 [Cs]}, \quad (3)$$

where,

$$\begin{aligned} T_g &= \text{the gas temperature (}^\circ\text{K)}, \\ \mu_o (Cs_2^+) &= \text{the molecular ion mobility at standard conditions} \\ &\quad (\text{cm}^2 / (\text{volt-sec.})), \\ [Cs_2^+] &= \text{the molecular ion concentration (cm}^{-3}\text{)}, \\ [Cs] &= \text{the cesium atom concentration (cm}^{-3}\text{)}, \\ \text{and } \Lambda &= \text{the characteristic diffusion length (cm)}. \end{aligned}$$

For a planar converter with interelectrode spacing,  $d(\approx \pi \Lambda)$ , equal to 0.5 cm, using  $T_g = 1400^\circ\text{K}$ ,  $[Cs_2^+]/[Cs] \approx [Cs_2^+]/[Cs]_o = 0.00269$  (a good approximation since the

degrees of ionization and excitation are shown to be small) and the Chanin and Steen value for the molecular ion mobility<sup>10</sup>,

$$\mu_o (Cs_2^+) = 0.21 \text{ cm}^2/(\text{volt-sec}), \text{ we find that}$$

$$D_a \frac{d^2 n}{dx^2} = 2.1 \times 10^{13} (\text{cm}^3\text{-sec})^{-1}.$$

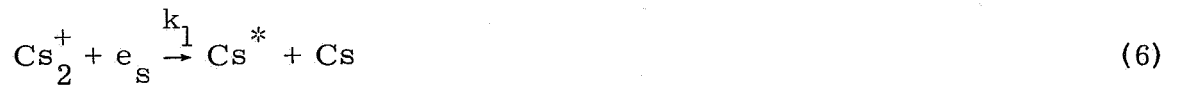
Under the above conditions except that  $d = 0.005\text{cm}$ , the diffusion rate is  $2.1 \times 10^{17} (\text{cm}^3\text{-sec})^{-1}$ . The former conditions apply to the Bullis and Wiegand<sup>11</sup> converter and it is seen that the diffusion term is insignificant in the electron balance equation 2. The latter conditions approximate those during experiments performed by V. C. Wilson<sup>12</sup>. Actually, the cesium atom density in the latter case was eight times that of the former, and any model used to analyze this close-spaced, high pressure converter would have to include the effects of ambipolar diffusion as well as those of radiation transport.

The model presented here is used to test to what extent radiation might control the particle concentrations in the plasma, to test the validity of neglecting charge transport in the volume and to test a set of kinetic equations to see if it suffices to display some essential features of a converter. It is not, however, the intent of the authors to use these calculations as the basis for a conclusion regarding the dominant positive ion present in converter plasmas.

### The Plasma Model

#### The Kinetic Equations

The following equations form the basis for the plasma model.



The excitation of cesium atoms from the  $6^2S$  to the  $6^2P$  state occurs upon collision of a fast electron with the ground state cesium atom as shown in equation 4( $k_x$ ).

Fast electrons are initially produced in the converter plasma as a result of the thermionically emitted electrons being accelerated by the emitter-sheath field as they enter the plasma. Although the 6P state is a doublet, only the  $6P_{3/2}$  state (1.45 eV, corresponding to a wave length of 8521 Å) will be considered here, since Freudenberg<sup>1</sup> observed that the intensity of the  $6P_{1/2}$  state (1.38 eV, corresponding to a wave length of 8943 Å) was small in comparison with the  $6P_{3/2}$  level intensity and in noting in addition that the amount of excitation energy is smaller, concluded that this energy level of the doublet plays only a secondary roll in the ionization process.

One method of quenching an excited atom is shown by equation 4; where  $k_{dx}$  denotes the de-excitation rate constant. One can see that in such a collision a slow electron (i. e. an electron whose energy is less than the first excitation potential of the cesium atom) is converted into a fast electron when the excitation energy of the atom is transferred to the electron. Thus, the reaction shown as equation 4 not only produces and quenches excited atoms but also transfers, in a rather discrete energy step, electrons between a high energy group and a low energy group. These are named fast and slow respectively for descriptive convenience.

The production of molecular ions occurs as shown in equation 5 ( $k_2$ ). No other ionization process is treated. Since the point of intersection of the repulsive portion of the  $Cs_2^{**}$  potential curve with that for  $Cs_2^+$  has not been determined, it is assumed that the bound, doubly-excited molecule is not a necessary precursor for molecular ion formation.

The products of recombination of the molecule ion with an electron must be consistent with the potential energy curves. Those proposed by DeBoer<sup>2</sup> and substantiated elsewhere<sup>13</sup> make  $k_3$  for the reverse reaction 5 very small under converter conditions. The dominant dissociative recombination reaction is given by equation 6.

In their microwave cavity experiment, Dandurand and Holt<sup>14</sup> observed that the molecular band spectrum which was emitted by the cesium plasma after termination of the electrodeless discharge was in the same wavelength range as previously found in the absorption measurements of Loomis and Kusch<sup>15</sup>. In fact, they found that the spectrum in the afterglow was almost entirely a band spectrum. Further, Rizzo and Bell<sup>4</sup> found that "emission bands adjacent to the primary atomic resonance line of cesium are the dominant band structure and appear to be excited directly by the trapped photons of the atomic resonance transition." These facts lead to the inclusion of the frequency-shifting reaction as shown by equation 7. This reaction is a ternary

collision, and the most probable one is with a  $\text{Cs}_2$  (when the cesium gas temperature is such to allow a reasonable molecule density) since the third body is readily available.

Equation 8 indicates that resonance radiation may be spontaneously emitted by an excited state while equation 9 shows that non-resonant radiation is emitted when the excited specie is a molecule.

### Radiation Transport

Holstein<sup>16</sup> treats the transmission of resonance radiation in gases and points out the error that K. T. Compton<sup>17</sup> and Milne<sup>18</sup> made when they assumed a uniform absorption coefficient and tried to treat excitation transfer as though the quanta diffused through the gas with a definite mean free path. The monochromatic transmission probability,  $T(\nu, \rho)$ , must be averaged over the frequency spectrum  $P(\nu)$ , to account for the rapid variation of the absorption coefficient,  $k(\nu)$ , with frequency  $\nu$ . Holstein writes the probability of radiation traversing a distance  $\rho$ ,  $T(\rho)$ , as

$$T(\rho) = \int_{-\infty}^{\infty} P(\nu) \exp(-k(\nu)\rho) d\nu, \quad (10)$$

rather than  $\exp(-k\rho)$  as Compton and Milne did.

C. Gregory<sup>19</sup> determined that resonance broadening of the  $8521\text{\AA}$  line obeys the dispersion relationship<sup>20</sup>,

$$k(\nu) = \frac{k_d}{1.0 + (4\pi(\nu - \nu_0)/\gamma_p)^2} \quad (11)$$

out to  $20\text{\AA}$  from the line center,  $\nu_0$ . Here,

$$k_d = \frac{\lambda_o^2 N}{2\pi} \frac{g_2}{g_1} \frac{\gamma}{\gamma_p} \quad (12)$$

where

$N$  = the density of normal atoms ( $\text{cm}^{-3}$ ),

$\lambda_o$  = the wavelength of the resonance line ( $8521\text{\AA}$ ),

$g_{1,2}$  = the statistical weight of the normal (excited) state,

$\gamma$  = the reciprocal of the lifetime ( $3.3 \times 10^{-8}$  sec) of the excited state

and  $\gamma_p = \frac{4}{3} \frac{\lambda_o e^2}{mc} fN$ ,

where  $e$  and  $m$  are the charge and mass of the electron and the  $f$ -value or oscillator strength is discussed by Mitchell and Zemansky<sup>21</sup>. The effects of hyperfine structure are omitted in this expression for  $\gamma_p$ .

Under the assumption that  $P(\nu) \propto k(\nu)$ , we have from equations 10 and 11 that

$$T(\rho) = \int_{-\infty}^{\infty} \frac{\exp(-k_d \rho / (1.0 + y^2))}{1.0 + y^2} dy$$

$$= I_0(k_d \rho / 2) \exp(-k_d \rho / 2), \quad (13)$$

where  $I_0$  is the zero order modified Bessel function of the first kind. The asymptotic form of  $T(\rho)$  is ( $k_d \rho \gg 1.0$ )

$$T(\rho) = (1.0 / (\pi k_d \rho))^{1/2} (1.0 + \frac{1.0}{4k_d \rho} + \frac{9.0}{32k_d^2 \rho^2} + \dots) \quad (14)$$

For the 8521 Å resonance line,  $k_d = 1.10 \times 10^5 \text{ cm}^{-1}$ , and thus  $k_d \rho \gg 1.0$  for all  $\rho \gtrsim 10^{-3} \text{ cm}$ , which is the case here.

Defining  $G(\bar{r}', \bar{r}) d\bar{r}$  as the probability that a photon emitted at  $\bar{r}'$  will be absorbed in the volume element  $d\bar{r}$  at  $\bar{r}$ , Holstein shows that

$$G(\bar{r}', \bar{r}) = - \frac{1}{4\pi \rho^2} \frac{\partial T(\rho)}{\partial \rho} \quad (15)$$

Thus, 
$$\gamma \int_{\text{vol}} [Cs^*]_{\bar{r}'} G(\bar{r}', \bar{r}) d\bar{r}'$$

is the rate of absorption of photons in the volume  $d\bar{r}$  at  $\bar{r}$  due to the decay of excited cesium atoms at volume elements  $d\bar{r}'$ .  $[Cs^*]_{\bar{r}'}$  denotes the cesium excited atom density at  $\bar{r}'$ .

Using equation 14 in equation 15, integrating over the two co-ordinates parallel to the electrode surface and evaluating the constants we find that the one dimensional kernel,  $H(x', x)$ , is given by

$$H(x', x) = 2.82 \times 10^{-4} (|x' - x|)^{-3/2}$$

$$+ 1.15 \times 10^{-9} (|x' - x|)^{-5/2} + 1.39 \times 10^{-14} (|x' - x|)^{-7/2}$$

$$+ \dots \quad (16)$$

This kernel, when used in the one dimensional form of equation 16, determines the transport of radiation in the cesium plasma.

### The Rate Equations

From equations 4 to 9 we write the following rate equations (the square brackets denote concentration per  $\text{cm}^3$ ) for:

#### FAST ELECTRONS

$$\frac{d[e_f]}{dt} = k_{dx} [Cs^*] [e_s] - k_x [Cs] [e_f] - \frac{([e_f] v_f)}{4} E, C, \quad (17)$$

## SLOW ELECTRONS

$$\frac{d[e_s]}{dt} = k_x [Cs][e_f] - k_{dx} [Cs^*][e_s] - k_1 [Cs_2^+][e_s] + k_2 [Cs^*]^2, \quad (18)$$

## MOLECULAR IONS

$$\frac{d[Cs_2^+]}{dt} = k_2 [Cs^*]^2 - k_1 [Cs_2^+][e_s] - \left( \frac{[Cs_2^+]}{4} \bar{v}^+ \right) E, C, \quad (19)$$

## RESONANCE QUANTA

$$\theta \frac{d[h\nu_o]}{dt} = k_6 [Cs^*] (1.0 - \int H(x, x') dx') - k_7 [Cs][h\nu_o], \quad (20)$$

## NORMAL ATOMS

$$[Cs] = [Cs]_0 - 2[Cs_2^+] - [Cs^*],$$

## EXCITED ATOMS

$$\begin{aligned} \frac{d[Cs^*]}{dt} = & k_1 [Cs_2^+][e_s] - 2k_2 [Cs^*]^2 - k_{dx} [Cs^*][e_s] + k_x [Cs][e_f] \\ & - k_4 [Cs^*][Cs_2] + k_5 [Cs][Cs_2^*] + k_9 [Cs][h\nu_o] - k_6 [Cs^*] \\ & + k_6 \int [Cs^*]_{x'} H(x', x) dx' + S \delta(\Delta x_E) - \left( \frac{[Cs^*]}{4} \bar{v}^* \right) E, C \end{aligned} \quad (21)$$

and we add the condition of

## ELECTRICAL NEUTRALITY

$$[Cs_2^+] = [e_f] + [e_s]. \quad (22)$$

The last term in equations 17, 19 and 21 denotes particle loss to the electrodes, and derives from a Maxwellian distribution of velocities. The velocities appearing in these loss terms are given by

$$v_f = (2E_f/m)^{1/2}, \quad (23)$$

$$\bar{v}^* = (8kT_g/\pi M)^{1/2} \quad (24)$$

and

$$\bar{v}^+ = 0.707 \bar{v}^*.$$

$E_f$  is the energy of the fast electron group,  $m$  is the electron mass,  $T_g$  is the cesium gas temperature in the plasma and  $M$  is the mass of a cesium atom.

In equation 21,  $S$  denotes the strength of an external source which is creating excited atoms in the plasma volume. The source strength is determined by assuming that all emitted electrons are accelerated by the emitter sheath field to an energy of at least 1.45 eV and that each electron then produces an excited atom. These electrons do not appear in the above equations since the electron transport problem is not treated here. They may be thought of as flowing into the plasma, forming excited atoms through inelastic collisions with normal cesium atoms, moving toward the collector so that the net flow to each volume element is zero, and finally being collected on that surface.

$\delta(\Delta x_E)$  has the properties shown below,

$$\delta(\Delta x_E) = \begin{cases} 1.0 & \text{for } x \text{ within the width } \Delta x_E \\ 0 & \text{elsewhere} \end{cases}$$

and the width  $\Delta x_E$  is taken to be much smaller than the interelectrode spacing. The effects that various choices for  $\Delta x_E$  and  $S$  have on the density profiles will be shown.  $[Cs]_0$  is the no-reaction cesium atom concentration, and for  $T_g = 1426^\circ K$  and a cesium pressure of 0.38 Torr is equal to

$$[Cs]_0 = 4.106 \times 10^{15} \text{ atoms-cm}^{-3}.$$

Writing the rate constant  $k_4$  as,  $k_4 = \sigma_4 v_R$ , where  $\sigma_4$  is the cross section for the  $Cs^* - Cs_2$  collision and  $v_R$  is the relative velocity of the colliding species, we have that

$$\begin{aligned} k_4 [Cs^*] [Cs_2] &= \sigma_4 v_R [Cs^*] [Cs]^2 / 4.2 \times 10^{20} \\ &= 1.8 \times 10^{15} (\text{cm}^3 - \text{sec})^{-1}, \end{aligned}$$

since  $^{13} [Cs]^2 / [Cs_2] = 4.2 \times 10^{20} \text{ cm}^{-3}$ . In comparison, we have that

$$k_2 [Cs^*]^2 = (4 \times 10^{-10}) (4.1 \times 10^{13})^2 = 6.7 \times 10^{17} (\text{cm}^3 - \text{sec})^{-1},$$

so the terms containing  $k_4$  and  $k_5$  in equation 21 will be neglected.

In the steady state we use equations 19 and 20 in equation 21 and rearrange to get the following quadratic equation for the concentration of excited atoms.

$$\begin{aligned} k_2 [Cs^*]^2 + (k_{dx} [e_s] + k_6 \int H(x, x') dx' + (\frac{\bar{v}^*}{4})_{E, C}) [Cs^*] \\ + (\frac{\bar{v}^+}{4})_{E, C} [Cs_2^+] - k_x [Cs] [e_f] - S \delta(\Delta x_E) - k_6 \int [Cs^*]_{x'} H(x', x) dx' = 0. \end{aligned} \quad (25)$$

Except for the excited atom concentration appearing in the transport integral, all concentrations in equation 25 and in the following four equations are concentrations in the volume element centered at  $x$ . From equations 17, 18 and 19 we have, in the main plasma region, that

$$[e_s] = \frac{k_2 [Cs^*]^2}{k_1 [Cs_2^+]} \text{ and } [e_f] = \frac{k_{dx} [Cs^*] [e_s]}{k_x [Cs]}, \quad (26) \text{ and } (27)$$

while for the extremes of the plasma it is found that

$$[e_s] = (1.0 + \frac{\bar{v}^+}{v_f}) [Cs_2^+] \text{ and } [e_f] = (\frac{\bar{v}^+}{v_f}) [Cs_2^+] \quad (28) \text{ and } (29)$$

Equations 22, 25, 26 and 27 or equations 19, 25, 28 and 29 are numerically solved to give the spatial variation of excited atoms, slow and fast electrons and molecular ions across the interelectrode gap of the converter. Complete reflection of quanta at the electrode surfaces is assumed.

## The Modeled Plasma

The plasma of one of the Bullis and Wiegand converters<sup>11</sup> will be modeled since their probe measurements of this plasma are available.

Their converter operated with an emitter temperature  $T_E = 1426^\circ \text{ K}$ , cesium pressure  $P_{\text{Cs}} = 0.38 \text{ Torr}$ , output voltage  $V_O = 0.012 \text{ volts}$ , terminal current about 1.2 amperes for 1.27 square centimeters of emitter area and an estimated interelectrode spacing of 0.494 centimeters. For the computation, the interelectrode space is divided into nine discrete volume regions, the first centered 0.007 cm from the emitter surface and each subsequent one centered 0.06 cm further toward the collector. The volume elements are approximately 3.8 kinetic theory gas mean free paths apart. The details of the plasma division are shown in Figure 1.

## The Rate Constants

Solution of the equations requires knowledge of four rate constants ( $k_1, k_2, k_x, k_{dx}$ ), each rate constant depending upon the relative velocity of the colliding particles and upon the cross section for the collision. In addition,  $k_2$  and  $k_1$  depend upon the probability of autoionization during a  $\text{Cs}^* - \text{Cs}^*$  collision and upon the probability of dissociative recombination during a  $\text{Cs}_2^+ - e_s$  collision respectively.

Assuming that the cross section for the  $\text{Cs}^* - \text{Cs}^*$  collision is given by the atom-atom cross section<sup>22</sup>,  $110 \text{ \AA}^2$ , and that the probability of autoionization,  $P_{\text{ai}}$ , is unity, from equation 24 for  $T_g = 1426^\circ \text{ K}$  we have that  $\bar{v}^* = 4.77 \times 10^4 \text{ cm-sec}^{-1}$ , and estimate

$$k_2 = v_R \sigma_{xx} P_{\text{ai}} = 3.708 \times 10^{-10} \text{ cm}^3 - \text{sec}^{-1}. \quad (30)$$

Since neither the cross section nor the probability of autoionization is known exactly, it cannot be said whether equation 30 represents a maximum, minimum or mid-range value for  $k_2$ . A factor of ten larger and smaller than the estimated value is not an unreasonable range for  $k_2$ . The recent experimental results of Kniazzezh and Carabateas<sup>23</sup> indicate the product  $\sigma_{xx} P_{\text{ai}}$  may be less than  $0.2 \text{ \AA}^2$ . However, their neglect of electron quenching of excited atoms and of ion-electron recombination leaves this value, in the authors' opinion, in considerable doubt. Also, their cesium pressure was more than an order of magnitude less than that used in practical converters, making their conclusion, that the amount of molecular ion production in a converter plasma is negligible, poorly founded.

The theoretical expression, as given by Bates and Dalgarno<sup>24</sup>, for the rate constant associated with dissociative recombination cannot be used for cesium since its use requires a knowledge of the molecule ion and product potential curves. However, in view of the experimental work of Harris<sup>25</sup>, of Hammer and Aubrey<sup>26</sup>, and of Dandurand

and Holt<sup>14</sup>, this rate constant must fall in the range:  $10^{-9} \lesssim k_1 \lesssim 10^{-6} \text{ cm}^3\text{-sec}^{-1}$ .

The excitation and de-excitation rate constants are estimated using the cross sections for these reactions as determined by Witting<sup>27</sup>. It is assumed that the de-excitation cross section is constant at  $45 \text{ Å}^2$ , and that the initial slope of the excitation cross section is  $80 \times 10^{-16} \text{ cm}^2\text{-eV}^{-1}$  to comply with the use of 1.45 eV rather than 1.40 eV for the excitation energy of the cesium atom. Thus,

$$k_x = v_f (80 \times 10^{-16})(E_f - 1.45) \quad 1.45 \leq E_f \leq 2.20 \text{ eV}$$

and 
$$k_{dx} = v_s (45 \times 10^{-16}),$$

where  $v_s$ , given by equation 23 with  $E_f$  replaced by the energy of the slow electron group,  $E_s$ , is the slow electron velocity.

### Results

The slow and fast electron group energies,  $E_s$  and  $E_f$  respectively, are not determined by this analysis except for the restriction that  $E_s < 1.45 \text{ eV} < E_f$ . However, it is possible to appeal to experiment to determine a particular electron distribution function and to define rigorously the group energies from this distribution function. W. Wiegand<sup>28</sup> has indicated that the probe characteristics determine an electron energy distribution which does not differ significantly from a Maxwellian and that the average electron energy,  $kT_e$ , in the main ionizing region of the plasma is approximately equal to  $2kT_E$ , where  $T_E$  is the emitter temperature. The assumption of a Maxwellian distribution lends itself nicely to the two group calculations of the model since 1.45 eV is a natural cut-off energy for defining the slow and fast electron groups. The group energies are defined by

$$E_s = \frac{\int_0^{E_0} M(E)E dE}{\int_0^{E_0} M(E) dE} \quad \text{and} \quad E_f = \frac{\int_{E_0}^{\infty} M(E)E dE}{\int_{E_0}^{\infty} M(E) dE}, \quad (31) \text{ and } (32)$$

where  $E_0 = 1.45 \text{ eV}$  and  $M(E)dE$  is the Maxwellian distribution in energy space. With  $1/2mv_p^2 = kT_e = 0.25 \text{ eV}$ , evaluation of the integrals in equations 31 and 32 yields  $E_s = 0.363 \text{ eV}$  and  $E_f = 1.717 \text{ eV}$ . Using these group energies we have that  $k_x = 1.66 \times 10^{-7} \text{ cm}^3\text{-sec}^{-1}$  and  $k_{dx} = 1.60 \times 10^{-7} \text{ cm}^3\text{-sec}^{-1}$ . Since an energy distribution has been assumed, it is possible to determine the ratio  $k_1:k_2$  for a given  $k_1$  as follows. The denominators of equations 31 and 32 give the fraction of electrons that are in the slow and fast groups respectively, and the two group approximation merely places all the electrons in each of these groups at one particular energy. Thus, if  $k_1$  is chosen, there is only one value of  $k_2$  which predicts the required group concentrations. For the

Maxwellian distribution at  $kT_e = 0.25$  eV, these fractions are  $[e_s]/[Cs_2^+] = 0.99111$  and  $[e_f]/[Cs_2^+] = 0.00889$  at the point of maximum electron density.

Figure 2 shows the results of varying  $k_2$  ( $k_1$  constant), and the intersection of each of the curves with the horizontal line,  $[e_s]/[Cs_2^+] = 0.99111$ , yields the value of  $k_2$  that satisfies the above concentration ratios. These intersection points determine the curve of Figure 3 and are tabulated in Table 1 along with the corresponding  $Cs^*$  and  $Cs_2^+$  concentrations at 0.127 cm from the emitter surface. The last column of Table 1 contains the probe-determined value for the electron concentration at  $x = 0.127$  cm. The corresponding molecular ion density variations across the interelectrode gap are shown in Figure 4. For these calculations the source,  $S_0(\Delta x_E)$ , was distributed equally between the volumes centered at  $x = 0.127$  and  $x = 0.187$  cm from the emitter surface (see Figure 1). Perhaps all of the emitted electrons do not gain sufficient energy for excitation as they are accelerated into the plasma, in which case, the charge densities would be less than shown in Figure 4. However, from Figure 5 it is seen that a 70% decrease in the source strength causes only a 30% decrease in the maximum  $Cs_2^+$  concentration. The fraction of slow electrons at the point of maximum charge concentration is not a constant along the curve of Figure 5, and the variation of this fraction with source strength is shown in Figure 6. Referring to Figure 2, we estimate that with  $S = 0.3S_0$  and  $k_2 = 6.1 \times 10^{-10} \text{ cm}^3\text{-sec}^{-1}$ ,  $k_1$  would have to be approximately  $2.0 \times 10^{-8} \text{ cm}^3\text{-sec}^{-1}$  to obtain the desired slow electron concentration. This value for  $k_1$  is in good agreement with the experimental results of Harris<sup>25</sup> and of Hammer and Aubrey<sup>26</sup>.

Figure 5 shows the variation of  $Cs_2^+$  across the interelectrode gap under the assumption of the equal distribution of input fast electrons as discussed above (50-50 distribution), and under the assumption that two-thirds of the excitation occurs in the volume nearest the emitter surface, the remaining one-third in the volume element centered around  $x = 0.187$  cm (67-33 distribution). The total source strength, based upon the measured current of 1.2 amperes for the  $1.27 \text{ cm}^2$  emitter surface is  $5.90 \times 10^{18} (\text{cm}^3\text{-sec})^{-1}$ . It is seen that the choice of a source distribution affects essentially only the density in the main volume ionization region.

### Conclusions

A model of the cesium plasma existing in the interelectrode gap of an arc mode converter has been investigated. The various plasma specie concentrations are determined by the radiation field and photon transport is the only transport considered in the main volume of the plasma. Quantitative agreement with experiment is not investigated since many of the important cross sections have not been either experimentally

determined or theoretically predicted. This fact causes the rate constants to be unknown, and meaningful comparison cannot be made.

However, the model shows that radiation transport can determine reasonable spatial distributions in the plasma, and for reasonable values of the rate constants it exhibits qualitatively some of the experimentally observed behavior. The model also indicates that the molecular ion is a probable ionic specie as long as the ratio  $k_1/k_2$  is not very large (see Table 1 and Figure 5). In particular, it is shown that a large value of  $k_1$  (e. g. the Dandurand and Holt<sup>14</sup> experimental value) is neither compatible with a reasonable value for  $k_2$  nor with a molecule ion density comparable to that measured by Bullis and Wiegand. If the inequality,  $k_1 \gg k_2$ , exists, molecular ions recombine at a rate so much greater than their rate of formation that a significant steady state density cannot be maintained. One then looks to step-wise excitation of the cesium atom and the subsequent production of  $\text{Cs}^+$  ions in the plasma to explain the neutralization of space charge<sup>29, 30</sup>.

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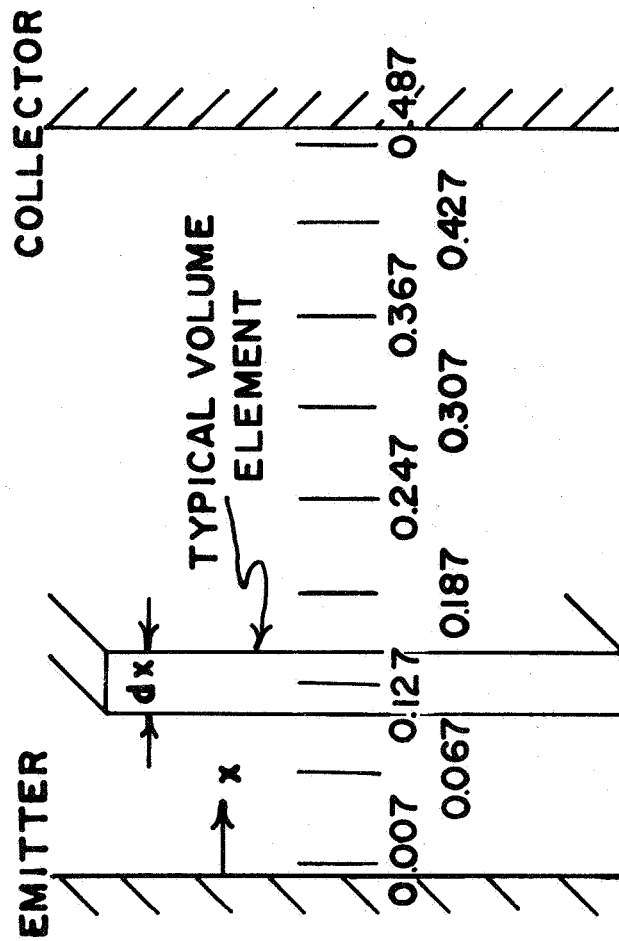
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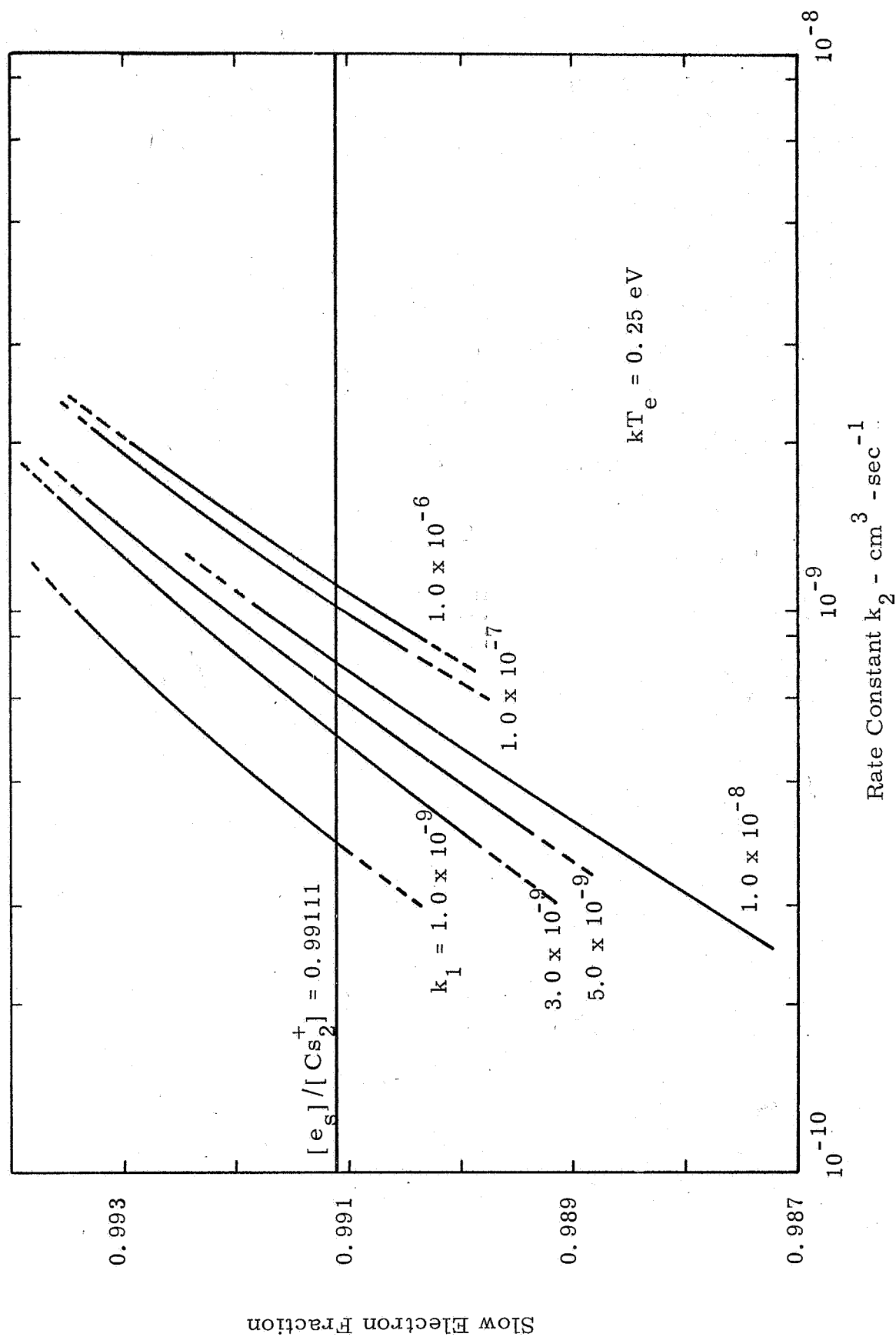
$k_1$ (cm <sup>3</sup> -sec <sup>-1</sup> )	$k_2$ (cm <sup>3</sup> -sec <sup>-1</sup> )	$k_1/k_2$	$[Cs^*]/[Cs]_0$	$[Cs_2^+]/[Cs]_0$	$n_e/[Cs]_0$
$1.0 \times 10^{-6}$	$1.13 \times 10^{-9}$	885	0.931	0.0315	0.336
$1.0 \times 10^{-7}$	$1.04 \times 10^{-9}$	96.1	0.935	0.0956	0.336
$1.0 \times 10^{-8}$	$8.10 \times 10^{-10}$	12.3	0.947	0.269	0.336
$5.0 \times 10^{-9}$	$7.20 \times 10^{-10}$	6.94	0.925	0.350	0.336
$3.0 \times 10^{-9}$	$6.10 \times 10^{-10}$	4.92	0.916	0.412	0.336
$1.0 \times 10^{-9}$	$3.90 \times 10^{-10}$	2.56	0.907	0.572	0.336

TABLE 1 - MAXIMUM EXCITED ATOM AND MOLECULAR ION DENSITIES IN THE PLASMA AS PREDICTED BY THE MODEL. ELECTRONS ARE ASSUMED TO HAVE A MAXWELLIAN DISTRIBUTION OF ENERGIES WITH  $kT_e = 0.25$  eV. CONCENTRATION RATIOS ARE EXPRESSED IN PER CENT WITH  $[Cs]_0 = 4.106 \times 10^{15}$  cm<sup>-3</sup>.



**FIGURE 1 - THE INTERELECTRODE PLASMA; SHOWING THE NINE SPACE POINTS USED FOR COMPUTATIONAL PURPOSES; DISTANCES SHOWN ARE IN CENTIMETERS.**

FIGURE 2 - FRACTION OF PLASMA ELECTRONS THAT ARE SLOW  
VERSUS  
THE RATE CONSTANT  $k_2$ , FOR FIXED VALUES OF THE RATE CONSTANT  $k_1$ .



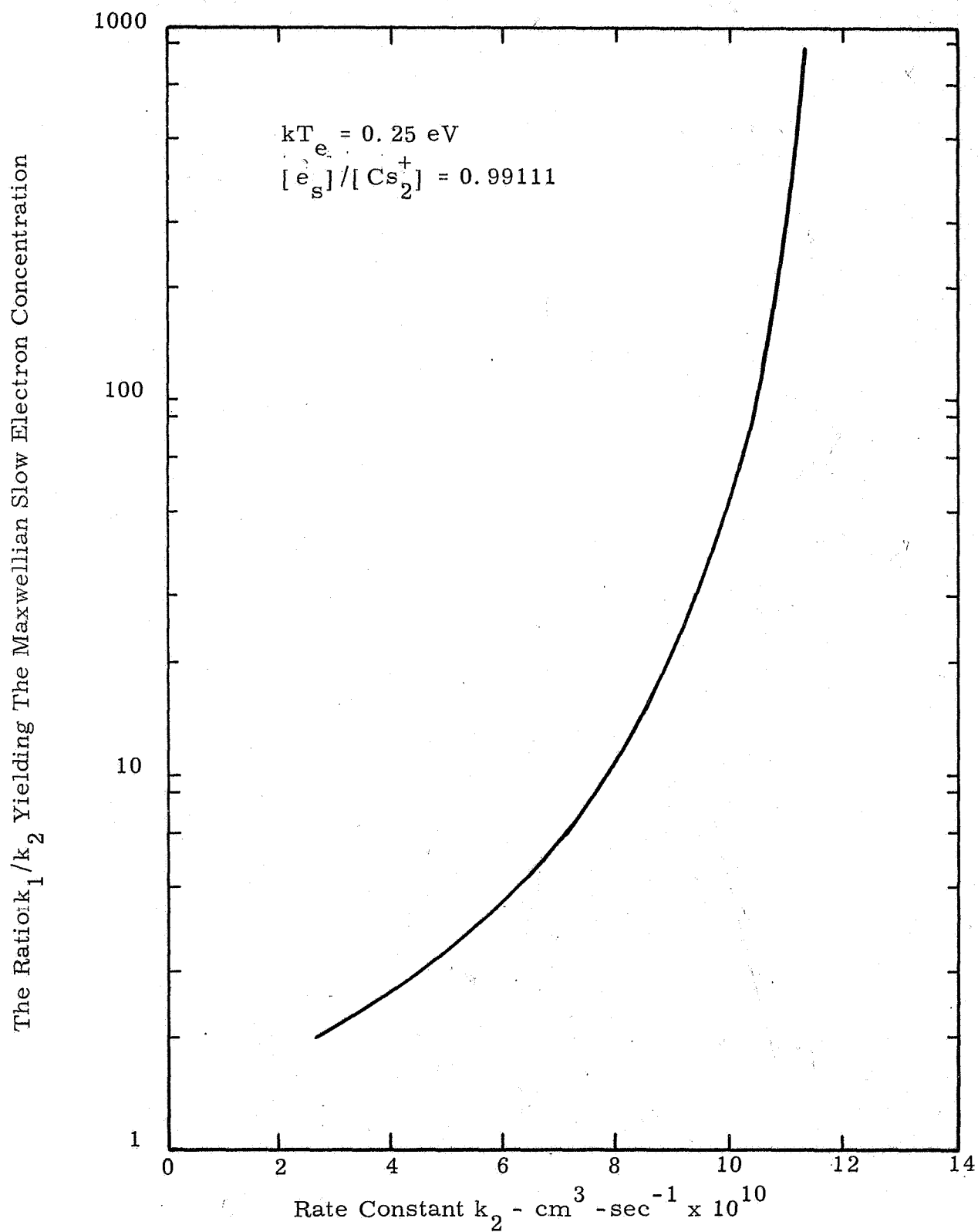
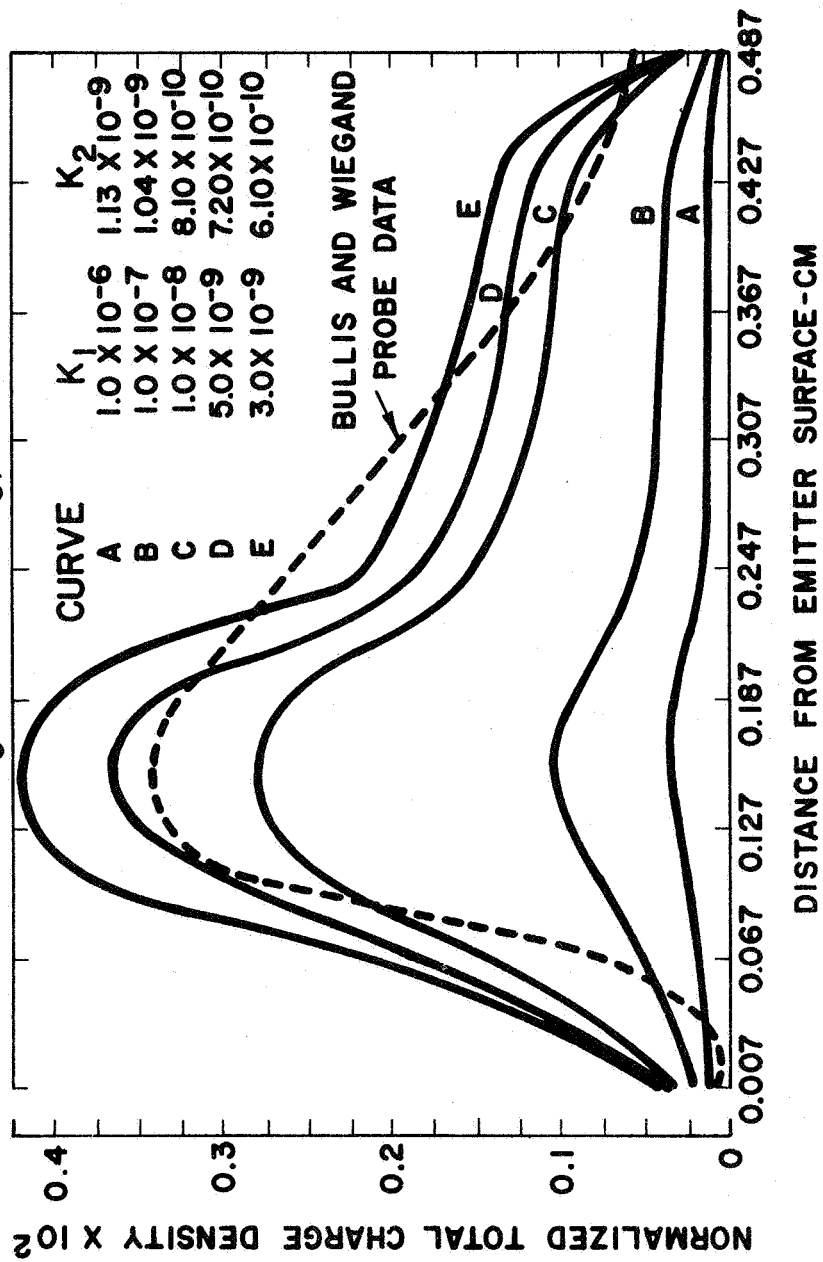


FIGURE 3 - THE RATE CONSTANT  $k_2$  VERSUS THE RATIO OF RATE CONSTANTS  $k_1/k_2$  FOR A MAXWELLIAN ELECTRON DISTRIBUTION.

FIGURE 4 - MOLECULAR ION DENSITY ACROSS THE  
 INTERELECTRODE CESIUM PLASMA, SHOWING  
 THE DEPENDENCE UPON THE RATE CONSTANTS  
 $K_1$  AND  $K_2$ .  $KT_e = 0.25\text{eV}$   $[e_s]/[Cs^+] = 0.9911$



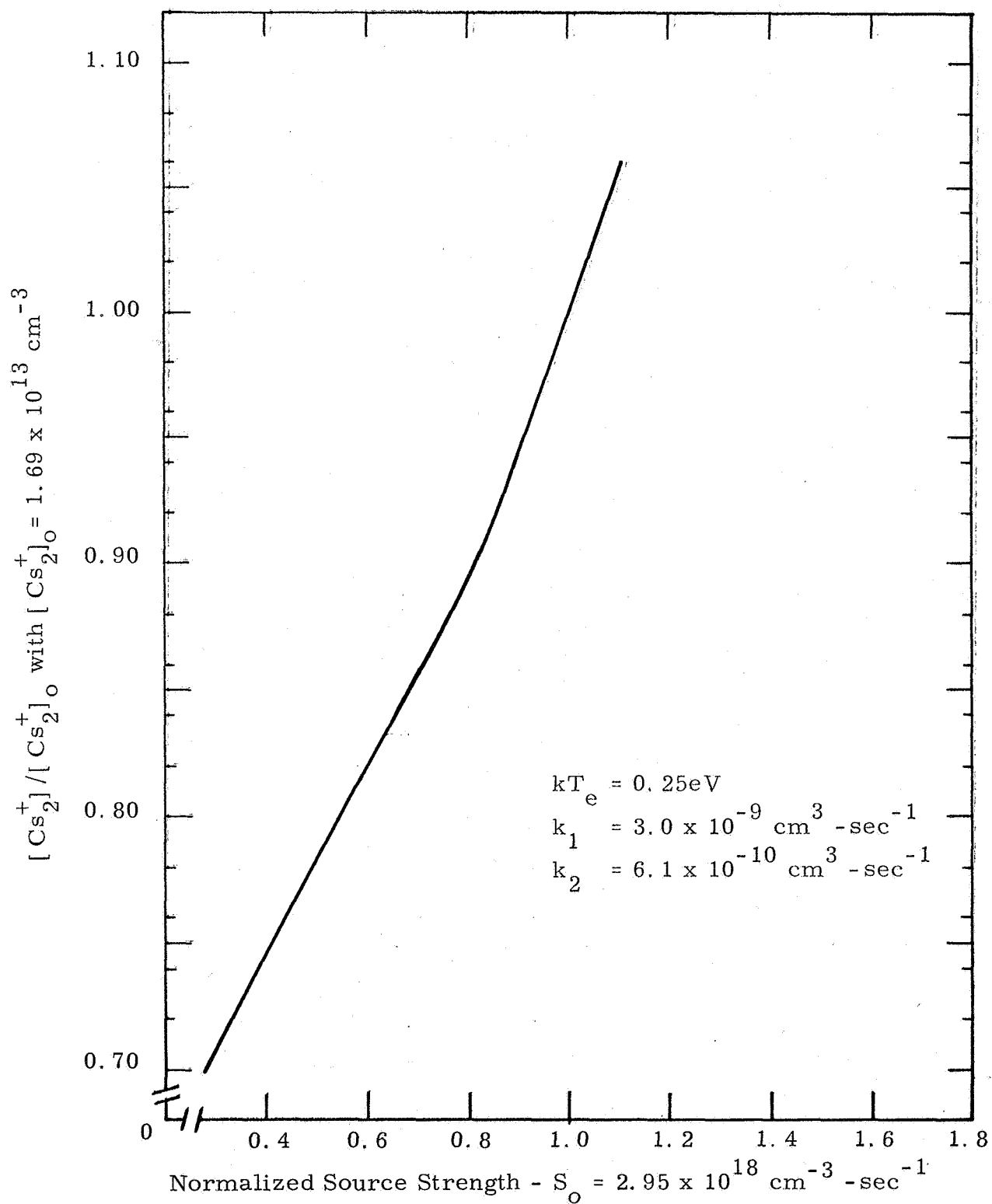


FIGURE 5 - MOLECULAR ION CONCENTRATION AS A FUNCTION OF  
 THE EXCITED ATOM INPUT SOURCE STRENGTH  
 DISTANCE FROM THE EMITTER SURFACE IS 0.127 cm.

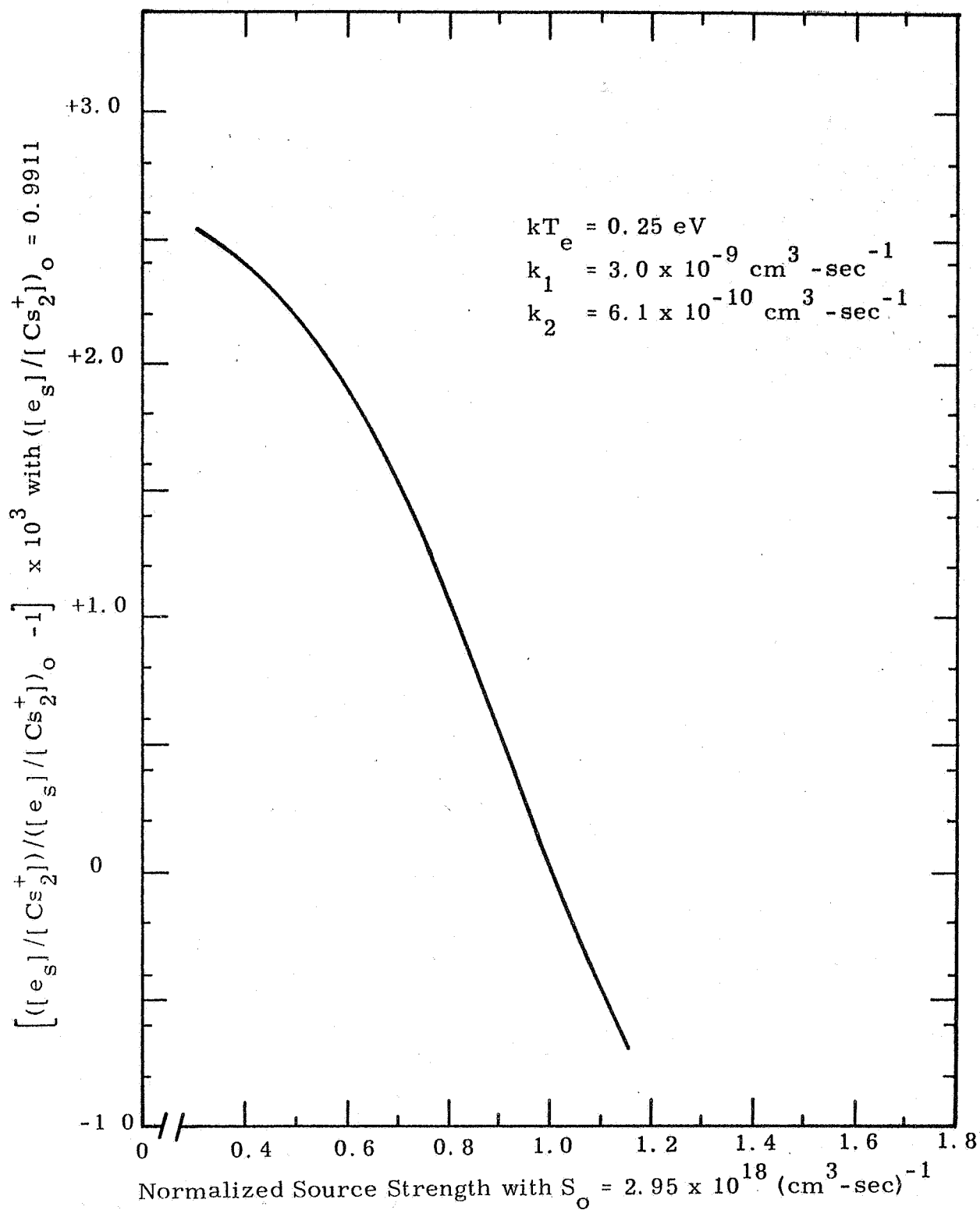


FIGURE 6 - SLOW ELECTRON FRACTION AS A FUNCTION OF THE  
 EXCITED ATOM INPUT SOURCE STRENGTH  
 DISTANCE FROM THE EMITTER SURFACE IS 0.127 cm.

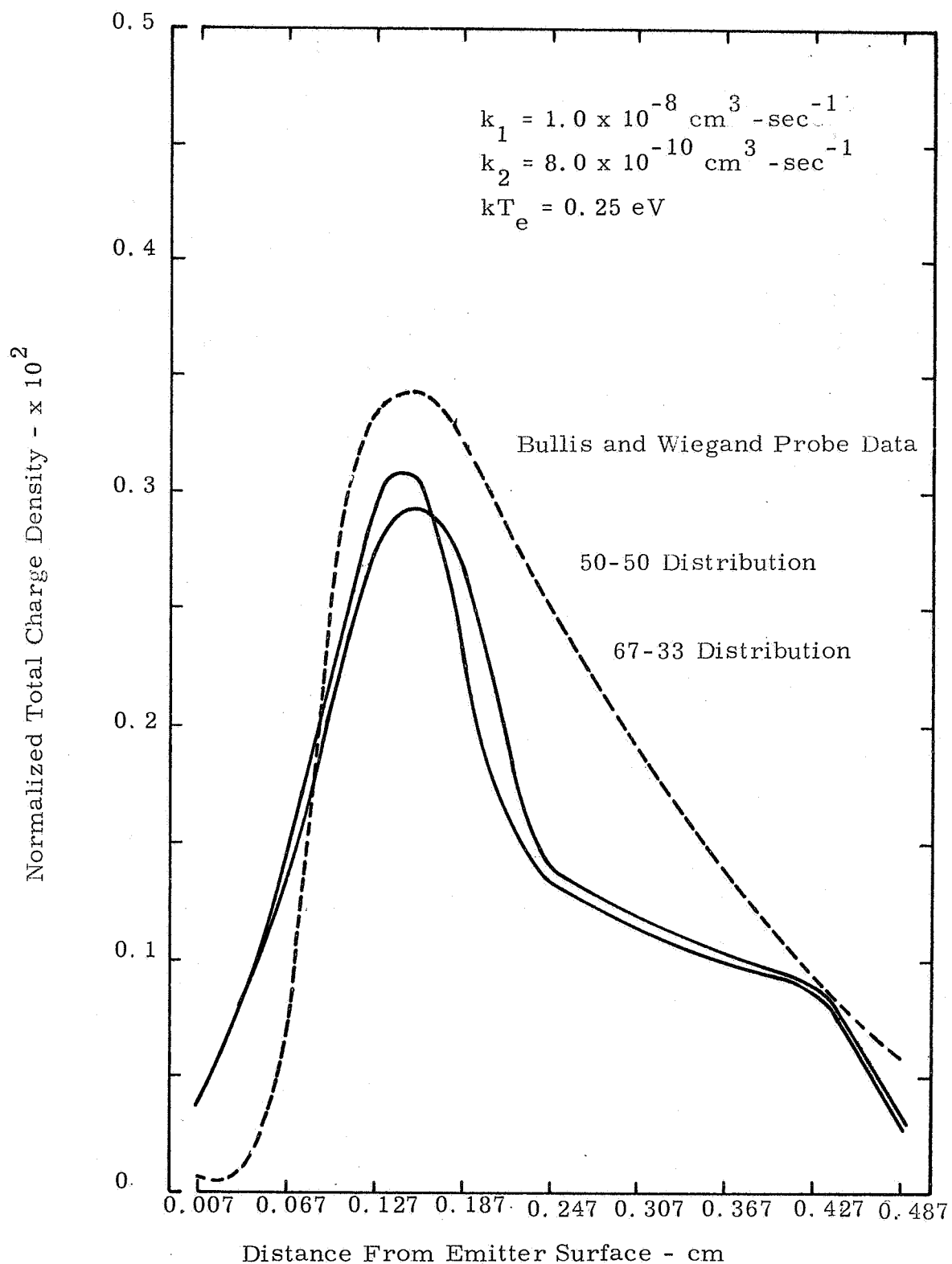


FIGURE 7 - MOLECULAR ION DENSITY ACROSS THE INTERELECTRODE CESIUM PLASMA, SHOWING THE EFFECTS OF DIFFERENT INPUT-SOURCE DISTRIBUTIONS